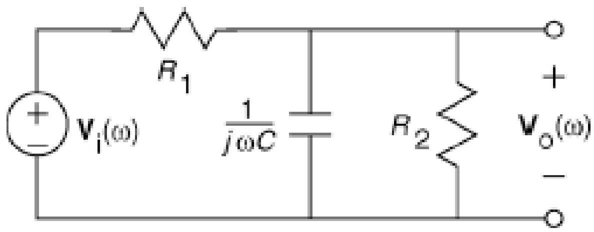
**P13.3-1**




$$R_2 \parallel \frac{1}{j\omega C} = \frac{R_2}{1+j\omega C R_2}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\frac{R_2}{1+j\omega C R_2}}{R_1 + \frac{R_2}{1+j\omega C R_2}} = \frac{\frac{R_2}{R_1+R_2}}{1+j\omega C R_p}$$

where $R_p = R_1 \parallel R_2$.

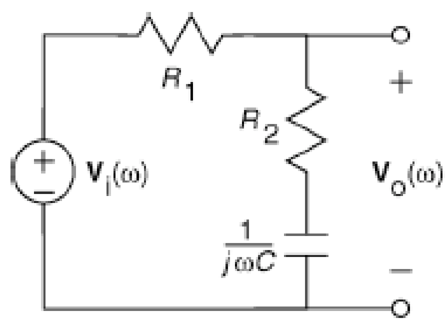
When $R_1 = 40\ \Omega$, $R_2 = 10\ \Omega$ and $C = 0.5$ F

$$\mathbf{H}(\omega) = \frac{0.2}{1+j4\omega}$$

(checked using ELab on 8/6/02)

**P13.3-2**

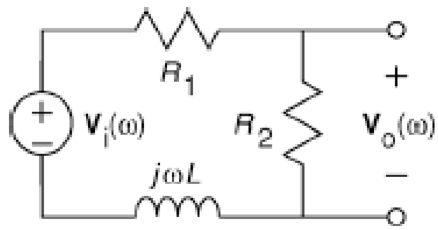
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{R_2 + \frac{1}{j\omega C}}{R_1 + R_2 + \frac{1}{j\omega C}} = \frac{1+j\omega C R_2}{1+j\omega C\left(R_1+R_2\right)}$$




When $R_1 = 40\ \text{k}\Omega$, $R_2 = 160\ \text{k}\Omega$ and $C = 0.025\ \mu\text{F}$

$$\mathbf{H}(\omega) = \frac{1+j(0.004)\omega}{1+j(0.005)\omega}$$

**P13.3-3**



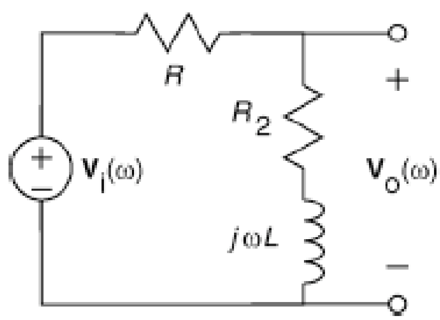

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{R_2}{R_1 + R_2 + j\omega L} = \frac{\dfrac{R_2}{R_1 + R_2}}{1 + j\omega \dfrac{L}{R_1 + R_2}}$$

When $R_1 = 4\ \Omega$, $R_2 = 6\ \Omega$ and $L = 8$ H

$$\mathbf{H}(\omega) = \frac{0.6}{1 + j(0.8)\omega}$$

(checked using ELab on 8/6/02)

**P13.3-4**



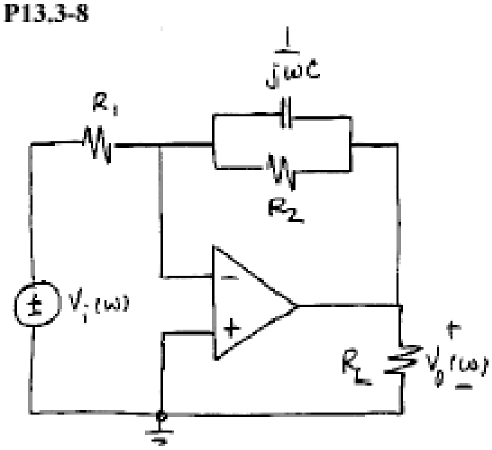

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{R_2 + j\omega L}{R + R_2 + j\omega L} = \left(\frac{R_2}{R + R_2}\right)\left(\frac{1 + j\omega \dfrac{L}{R_2}}{1 + j\omega \dfrac{L}{R + R_2}}\right)$$

Comparing the given and derived network functions, we require

$$\left(\frac{R_2}{R + R_2}\right)\left(\frac{1 + j\omega \dfrac{L}{R_2}}{1 + j\omega \dfrac{L}{R + R_2}}\right) = (0.6)\frac{1 + j\dfrac{\omega}{12}}{1 + j\dfrac{\omega}{20}} \Rightarrow \begin{cases} \dfrac{R_2}{R + R_2} = 0.6 \\ \dfrac{R_2}{L} = 12 \\ \dfrac{R + R_2}{L} = 20 \end{cases}$$

Since $R_2 = 60\ \Omega$, we have $L = \frac{60}{12} = 5\ \text{H}$, then $R = (20)(5) - 60 = 40\ \Omega$.

**P13.3-8**



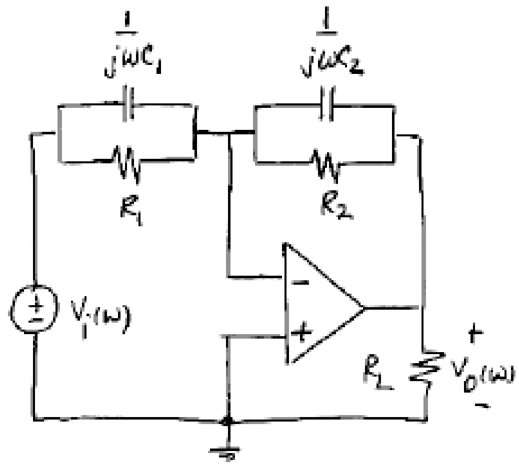

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}=-\frac{R_2 \parallel \dfrac{1}{j\omega C}}{R_1}$$

$$=\frac{-\left(\dfrac{R_2}{R_1}\right)}{1+j\omega C\,R_2}$$

When $R_1=10\text{ k}\Omega$, $R_2=50\text{ k}\Omega$, and $C=2\ \mu\text{F}$, then

$\frac{R_2}{R_1}=5$ and $R_2C=\frac{1}{10}$ so $\mathbf{H}(\omega)=\frac{-5}{1+j\frac{\omega}{10}}$

**P13.3-9**




$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}=-\frac{R_2 \parallel \dfrac{1}{j\omega C_2}}{R_1 \parallel \dfrac{1}{j\omega C_1}}$$

$$=-\frac{\dfrac{R_2}{1+j\omega\,C_2R_2}}{\dfrac{R_1}{1+j\omega C_1R_1}}$$

$$\mathbf{H}(\omega)=-\left(\frac{R_2}{R_1}\right)\left(\frac{1+j\omega C_1R_1}{1+j\omega C_2R_2}\right)$$

When $R_1=10\text{ k}\Omega$, $R_2=50\text{ k}\Omega$, $C_1=4\ \mu\text{F}$ and $C_2=2\mu\text{F}$,

then $\frac{R_2}{R_1}=5$, $C_1R_1=\frac{1}{25}$ and $C_2R_2=\frac{1}{10}$

so

$$\mathbf{H}(\omega)=-5\left(\frac{1+j\dfrac{\omega}{25}}{1+j\dfrac{\omega}{10}}\right)$$

$$gain = |\mathbf{H}(\omega)| = (5)\frac{\sqrt{1+\frac{\omega^2}{625}}}{\sqrt{1+\frac{\omega^2}{100}}}$$

$$phase\ shift = \angle \mathrm{H}(\omega) = 180 + \tan^{-1}\left(\frac{\omega}{25}\right) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

**P13.3-11**

$$\mathbf{H}(\omega) = -\frac{R_2 + \frac{1}{j\omega C}}{R_1} = -\frac{1 + j\omega\, C\, R_2}{j\omega C\, R_1}$$

$$\angle \mathbf{H}(\omega) = 180° + \tan^{-1} \omega C\, R_2 - 90°$$

$$\angle \mathbf{H}(\omega) = 135° \Rightarrow \tan^{-1} \omega C R_2 = 45° \Rightarrow \omega C\, R_2 = 1$$

$$\Rightarrow R_2 = \frac{1}{10^3 10^{-7}} = 10\ \text{k}\Omega$$

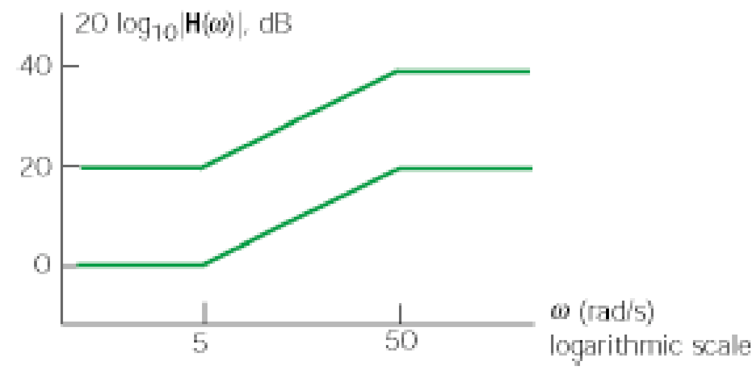
$$10 = \lim_{\omega \to \infty} |\mathbf{H}(\omega)| = \frac{R_2}{R_1} \Rightarrow R_1 = \frac{R_2}{10} = 1\ \text{k}\Omega$$

**P13.4-2**

$$\mathbf{H}_1(\omega) = \frac{1+j\dfrac{\omega}{5}}{1+j\dfrac{\omega}{50}} \qquad \mathbf{H}_2(\omega) = 10\,\frac{1+j\dfrac{\omega}{5}}{1+j\dfrac{\omega}{50}}$$

Both $\mathbf{H}_1(\omega)$ and $\mathbf{H}_2(\omega)$ have a pole at $\omega = 50$ rad/s and a zero at $\omega = 5$ rad/s. The slopes of both magnitude Bode plots increase by 20 dB/decade at $\omega = 5$ rad/s and decrease by 20 dB/decade at $\omega = 50$rad/s. The difference is that for $\omega < 5$rad/s

$$\left|\mathbf{H}_1(\omega)\right| \cong 1 = 0 \text{ dB} \quad \text{and} \quad \left|\mathbf{H}_2(\omega)\right| \cong 10 = 20 \text{ dB}$$



**P13.4-3**

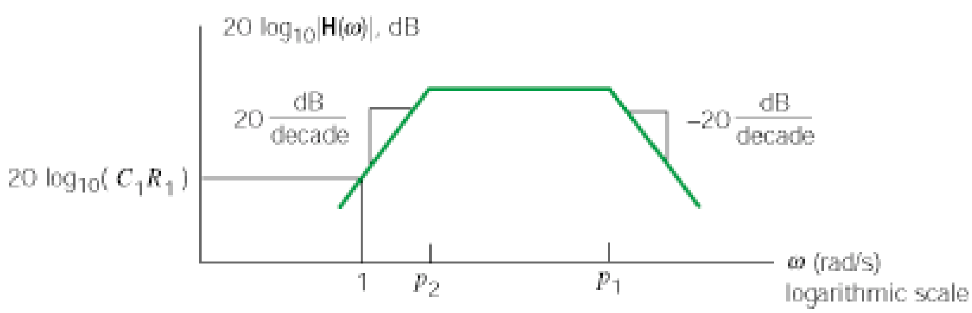
$$\mathbf{H}(\omega) = -\frac{\dfrac{R_2}{1+j\omega C_2 R_2}}{R_1+\dfrac{1}{j\omega C_1}} = -C_1 R_2 \frac{j\omega}{(1+j\omega R_1 C_1)(1+j\omega R_2 C_2)}$$

This network function has poles at

$$p_1 = \frac{1}{R_1 C_1} = 2000\text{ rad/s} \text{ and } p_2 = \frac{1}{R_2 C_2} = 1000\text{ rad/s}$$

so

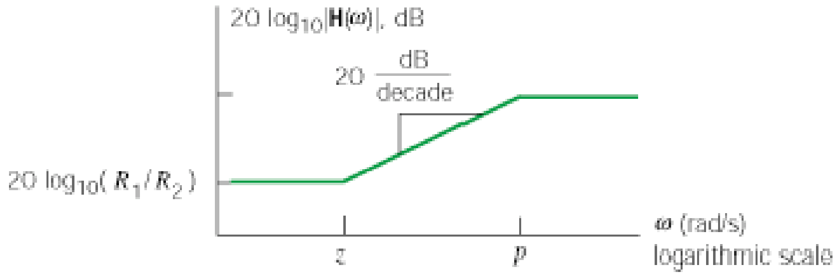
$$\mathbf{H}(\omega) \cong \begin{cases} (C_1 R_2)\, j\omega & \omega < p_1 \\ (C_1 R_2)\dfrac{j\omega}{j\omega C_1 R_1} = \dfrac{R_2}{R_1} = 2 & p_1 < \omega < p_2 \\ (C_1 R_2)\dfrac{j\omega}{(j\omega C_1 R_1)(j\omega C_2 R_2)} = \dfrac{1}{j\omega C_2 R_1} & \omega > p_2 \end{cases}$$



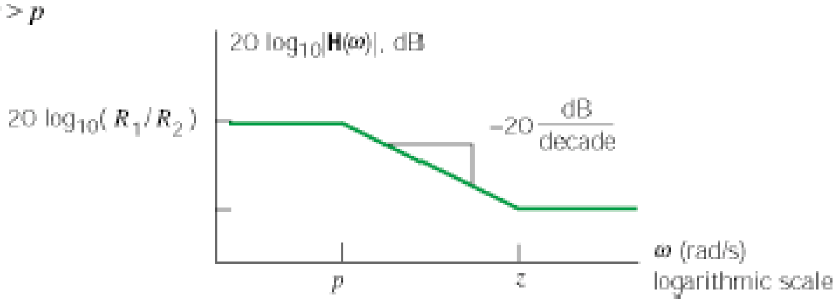
**P13.4-4**

$$\mathbf{H}(\omega) = -\frac{\dfrac{R_2}{1+j\omega C_2 R_2}}{\dfrac{R_1}{1+j\omega C_1 R_1}} = -\frac{R_2(1+j\omega C_1 R_1)}{R_1(1+j\omega C_2 R_2)} \text{ so } K = -\frac{R_2}{R_1},\ z = \frac{1}{C_1 R_1} \text{ and } p = \frac{1}{C_2 R_2}$$

When $z < p$




When $z > p$



**P13.5-1**

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(\frac{1}{120}\right)\left(\frac{1}{30}\ x\ 10^{-6}\right)}} = 60\ \text{k rad/sec}$$

$$Q = R\sqrt{\frac{C}{L}} = 10,000\sqrt{\frac{\frac{1}{30}\times 10^{-6}}{\frac{1}{120}}} = 20$$

$$\omega_1 = -\frac{\omega_0}{2Q} + \sqrt{\left(\frac{\omega_0}{2Q}\right)^2 + \omega_0^{\ 2}} = 58.52\ \text{k rad/s} \quad \text{and} \quad \omega_2 = \frac{\omega_0}{2Q} + \sqrt{\left(\frac{\omega_0}{2Q}\right)^2 + \omega_0^{\ 2}} = 61.52\ \text{k rad/s}$$

$$BW = \frac{1}{RC} = \frac{1}{(10000)\left(\frac{1}{30}\times 10^{-6}\right)} = 3\ \text{k rad/s}$$

Notice that $BW = \omega_2 - \omega_1 = \frac{\omega_0}{Q}$.

**P13.5-2**

$$\left|\mathbf{H}(\omega)\right| = \frac{k}{\sqrt{1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}}$$

so

$$R = k = \left|\mathbf{H}(\omega_0)\right| = \frac{8}{20\cdot 10^{-3}} = 400\ \Omega \quad \text{and} \quad \omega_0 = 1000\ \ \text{rad/s}$$

At $\omega = 897.6$ rad/s, $\left|\mathbf{H}(\omega)\right| = \frac{4}{20.10^{-3}} = 200$, so

$$200 = \frac{400}{\sqrt{1 + Q^2\left(\frac{897.6}{1000} - \frac{1000}{897.6}\right)^2}} \quad \Rightarrow \quad Q = 8$$

Then

$$\left.\begin{array}{l} \frac{1}{\sqrt{LC}} = \omega_0 = 1000 \\ 400\sqrt{\frac{C}{L}} = Q = 8 \end{array}\right\} \Rightarrow \begin{array}{l} C = 20\ \mu\text{F} \\ L = 50\ \text{mH} \end{array}$$

**P13.5-3**

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^5 \text{ rad/s}, \quad Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 10, \quad BW = \frac{R}{L} = 10^4 \text{ rad/s}$$