P14.3-1

$$\left.\begin{array}{r}\mathcal{L}\left[A\ f_1(t)\right]=A\ F_1(s)\\ f_1(t)=\cos(\omega t)\quad\Rightarrow\quad F_1(s)=\dfrac{s}{s^2+\omega^2}\end{array}\right\}\quad\Rightarrow\quad F(s)=\frac{As}{s^2+\omega^2}$$

P14.3-2

$$\mathcal{L}^{-1}\left[t^n\right]=\frac{n!}{s^{n+1}}\quad \mathrm{F}(s)=\mathcal{L}^{-1}\left[t^1\right]=\frac{1!}{s^{1+1}}=\frac{1}{s^2}$$

P14.3-3

Linearity: $\mathcal{L}\left[\mathrm{a}_1 f_1(t)+a_2 f_2(t)\right]=\mathrm{a}_1 F_1(s)+a_2 F_2(s)$

Here $a_1=a_2=1$

$$\mathcal{L}\left[f_1(t)\right]=\mathcal{L}\left[e^{-3t}\right]=\frac{1}{s+3}=F_1(s)$$

$$\mathcal{L}\left[f_2(t)\right]=\mathcal{L}[t]=\frac{1}{s^2}=F_2(s)$$

so $\underline{F(s)=\frac{1}{s+3}+\frac{1}{s^2}}$

P14.5-1

$$F(s)=\frac{s+3}{s^3+3s^2+6s+4}=\frac{s+3}{(s+1)\left[(s+1)^2+3\right]}=\frac{A}{s+1}+\frac{Bs+C}{s^2+2s+4}$$

where

$$A=\left.\frac{s+3}{(s+1)^2+3}\right|_{s=-1}=\frac{2}{3}$$

Then

$$\frac{(s+3)}{(s+1)\left(s^2+2s+4\right)}=\frac{\frac{2}{3}}{s+1}+\frac{Bs+C}{s^2+2s+4}\Rightarrow(s+3)=\left(\frac{2}{3}+B\right)s^2+\left(\frac{4}{3}+B+C\right)s+\frac{8}{3}+C$$

Equating coefficient yields

$$\mathrm{s}^2:\quad 0=\frac{2}{3}+B\quad\Rightarrow\quad B=-\frac{2}{3}$$

$$\mathrm{s}:\quad 1=\frac{4}{3}-\frac{2}{3}+C\quad\Rightarrow\quad C=\frac{1}{3}$$

Then

$$F(s)=\frac{\frac{2}{3}}{s+1}+\frac{-\frac{2}{3}s+\frac{1}{3}}{(s+1)^2+3}=\frac{\frac{2}{3}}{s+1}+\frac{-\frac{2}{3}(s+1)}{(s+1)^2+3}+\frac{\frac{1}{\sqrt{3}}\sqrt{3}}{(s+1)^2+3}$$

Taking the inverse Laplace transform yields

$$f(t)=\frac{2}{3}e^{-t}-\frac{2}{3}e^{-t}\cos\sqrt{3}t+\frac{1}{\sqrt{3}}e^{-t}\sin\sqrt{3}$$

P14.5-2

$$F(s)=\frac{s^2-2s+1}{s^3+3s^2+4s+2}=\frac{s^2-2s+1}{(s+1)(s+1-j)(s+1+j)}=\frac{a}{s+1-j}+\frac{a^*}{s+1+j}+\frac{b}{s+1}$$

where

$$b=\left.\frac{s^2-2s+1}{(s+1)^2+1}\right|_{s=-1}=4$$

$$a=\left.\frac{s^2-2s+1}{(s+1)(s+1+j)}\right|_{s=-1+j}=\frac{3-j4}{-2}=-\frac{3}{2}+j2$$

$$a^*=-\frac{3}{2}-j2$$

Then

$$F(s)=\frac{-\frac{3}{2}+j2}{s+1-j}+\frac{-\frac{3}{2}-j2}{s+1+j}+\frac{4}{s+1}$$

Next

$$m=\sqrt{(-3/2)^2+(2)^2}=\frac{5}{2}\quad\text{and}\quad\theta=\tan^{-1}\left(\frac{2}{-\frac{3}{2}}\right)=126.9°$$

From Equation 14.5-8

$$f(t)=\left[5e^{-t}\cos(t+127°)+4e^{-t}\right]u(t)$$

P14.5-3

$$F(s)=\frac{5s-1}{(s+1)^2(s-2)}=\frac{A}{s+1}+\frac{B}{(s+1)^2}+\frac{C}{s-2}$$

where

$$B=\left.\frac{5s-1}{s-2}\right|_{s=-1}=2\quad\text{and}\quad C=\left.\frac{5s-1}{(s+1)^2}\right|_{s=2}=1$$

Then

$$A=\left.\frac{d}{ds}\left[(s+1)^2F(s)\right]\right|_{s=-1}=\left.\frac{-9}{(s-2)^2}\right|_{s=-1}=-1$$

Finally

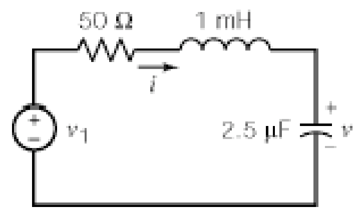
$$F(s)=\frac{-1}{s+1}+\frac{2}{(s+1)^2}+\frac{1}{s-2}\quad\Rightarrow\quad f(t)=\left[-e^{-t}+2te^{-t}+e^{2t}\right]u(t)$$

P14.6-1

(a) $$f(0) = \lim_{s \to \infty} sF(s) = \lim_{s \to \infty} \frac{2s^2 - 3s + 4}{s^2 + 3s + 2} = \frac{2s^2}{s^2} = 2$$

(b) $$f(\infty) = \lim_{s \to 0} sF(s) = \frac{4}{2} = 2$$

P14.7-1



KVL:

$$50i + 0.001\frac{di}{dt} + v = 2e^{-2\times10^4 t}$$

The capacitor current and voltage are related by

$$i = \left(2.5\times10^{-6}\right)\frac{dv}{dt}$$

$$v_1 = 2e^{-2\times10^4 t}\ \text{V},\quad i(0) = 1\ \text{A},\ v(0) = 8\ \text{V}$$

Taking the Laplace transforms of these equations yields

$$50\,I(s) + 0.001\left[s\,I(s) - i(0)\right] + V(s) = \frac{2}{s + 2\times10^4}$$

$$I(s) = \left(2.5\times10^{-6}\right)\left[sV(s) - v(0)\right]$$

Solving for I(s) yields

$$I(s) = \frac{s^2 + 1.4\times10^4 s - 1.6\times10^8}{\left(s+10^4\right)\left(s+2\times10^4\right)\left(s+4\times10^4\right)} = \frac{A}{s+10^4} + \frac{B}{s+2\times10^4} + \frac{C}{s+4\times10^4}$$

where

$$A = \left(s+10^4\right)I(s)\Big|_{s=-10^4} = \frac{s^2 + 1.4\times10^4 s - 1.6\times10^8}{\left(s+2\times10^4\right)\left(s+4\times10^4\right)}\Bigg|_{s=-10^4} = \frac{-2\times10^8}{3\times10^8} = \frac{-2}{3}$$

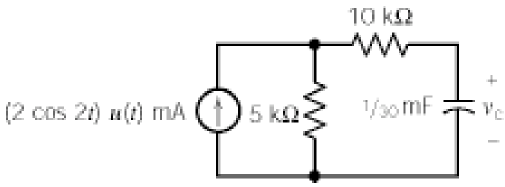
$$B = \left(s+2\times10^4\right)I(s)\Big|_{s=-2\times10^4} = \frac{s^2 + 1.4\times10^4 s - 1.6\times10^8}{\left(s+10^4\right)\left(s+4\times10^4\right)}\Bigg|_{s=-2\times10^4} = \frac{.4\times10^8}{2\times10^8} = \frac{1}{5}$$

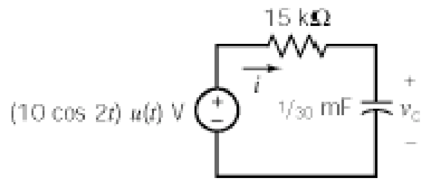
$$C = \left(s+4\times10^4\right)I(s)\Big|_{s=-4\times10^4} = \frac{s^2 + 1.4\times10^4 s - 1.6\times10^8}{\left(s+10^4\right)\left(s+2\times10^4\right)}\Bigg|_{s=-4\times10^4} = \frac{8.8\times10^8}{6\times10^8} = \frac{22}{15}$$

Then

$$I(s) = -\frac{2/3}{s+10^4} + \frac{1/5}{s+2\times10} + \frac{22/15}{s+4\times10^4} \Rightarrow i(t) = \frac{1}{15}\left[-10e^{-10^4 t} + 3e^{-2\times10^4 t} + 22e^{-4\times10^4 t}\right]u(t)\ \text{A}$$

P14.7-3





$v_C(0) = 0$

$$\left.\begin{array}{r} v_c + 15i = 10\cos 2t \\ i = \dfrac{1}{30}\dfrac{d\,v_c}{dt} \end{array}\right\} \Rightarrow \quad \frac{d\,v_c}{dt} + 2\,v_c = 20\cos 2t$$

Taking the Laplace Transform yields:

$$sV_c(s) - v_c(0) + 2V_c(s) = 20\frac{s}{s^2+4} \quad \Rightarrow \quad V_c(s) = \frac{20s}{(s+2)(s^2+4)} = \frac{A}{s+2} + \frac{B}{s+j2} + \frac{B^*}{s-j2}$$

where

$$A = \left.\frac{20s}{s^2+4}\right|_{s=-2} = \frac{-40}{8} = -5,\ B = \left.\frac{20s}{(s+2)(s-j2)}\right|_{s=-j2} = \frac{j5}{1+j} = \frac{5}{2} + j\frac{5}{2} \text{ and } B^* = \frac{5}{2} - j\frac{5}{2}$$

Then

$$V_c(s) = \frac{-5}{s+2} + \frac{\frac{5}{2}+j\frac{5}{2}}{s+j2} + \frac{\frac{5}{2}-j\frac{5}{2}}{s-j2} \quad \Rightarrow \quad v_c(t) = -5e^{-2t} + 5(\cos 2t + \sin 2t)\text{ V}$$